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RESEARCH ARTICLE

Estimation of Aspect Ratio Values in Carbonate Rocks Using the Kuster–Toksöz Model for Reservoir Characterization

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Abstract

Carbonate reservoirs are known for their complex pore structures, which significantly affect their elastic properties and, consequently, their fluid storage and flow capacities. Traditional models often fail to capture the heterogeneous pore geometries in carbonate rocks accurately. The Kuster–Toksöz model offers a means to estimate effective elastic moduli by incorporating pore aspect ratios, providing a more nuanced understanding of pore shapes and distributions The combination of Kuster-Toksöz inclusion model for pore shape of penny cracks produces velocity model which is then compared with the measurement results. The results show a good match at 2800 m - 3000 m intervals with a single aspect ratio of 0.05. The difference in the aspect ratio of each depth is related to the compacting process of rock formation due to increased depth, which shows that the deeper the depth the rock pore will be stiffer.

Keywords: Carbonate Reservoir, Inclusion Model, Kuster-Toksöz

1. Introduction

To get reliable results for reservoir characterization, a good understanding of the parameters in its reservoir is needed. Seismic data has a sensitivity to the subsurface depiction that is needed to get a lot of information about these parameters. But this has not completely eliminated the uncertainty in detecting the presence of hydrocarbons. So, we need an in-depth understanding by calculating the target reservoir rock properties (Schön, 2011).

Carbonate rocks are the most promising rock reservoir hosts in producing hydrocarbons throughout the world. However, carbonate rocks have a very complex pore system compared to clastic rocks, which are influenced by many processes such as dissolution, precipitation, dolomitization and others (Mavko et al., 2009). Rock physics studies involving the effects of pore type to predict and model the elastic property of carbonates have been carried out by many geoscientists. In most cases, pores in carbonates are often modeled as elliposidal inclusions from their aspect ratio (Li et al., 2018; Zhao et al., 2013). By using the inclusion model introduced by Kuster and Toksöz in 1974, an effective modulus was obtained from the inclusion composite media with different pore geometries in a background host material (Kuster and Toksöz, 1974). To model the saturation conditions of fluid the Gassmann equation can be used. Various other approaches and methods can also be used to support modeling of carbonate rock properties, although most approaches tend to be designed for clastic rocks. One example is to determine the aspect ratio of the Zimmermann equation which helps to see quickly the tendency of data behavior (Zimmerman, 1991). A combination of many approaches continues to be done to get maximum results in estimating various carbonate rock properties in accurately characterizing the reservoir (Hilman and Winardhi, 2019).

Despite these advances, previous studies generally treat pore aspect ratio as a fixed or assumed value, and only a limited number of works evaluate how aspect ratio varies with depth or compaction in carbonate reservoirs. Furthermore, the integration of Zimmerman pore stiffness analysis as a constraint for selecting aspect ratio inputs has rarely been combined systematically with the Kuster–Toksöz inclusion model and inverse Gassmann calculations. This creates a gap in understanding how pore geometry evolves vertically and how it affects elastic responses when calibrated directly with well-log velocities. Therefore, there is a need for a modeling workflow that links pore stiffness trends, dry-frame modulus estimation, and inclusion modeling to quantify depth-dependent pore aspect ratio variations in carbonate rocks.

The objective of this study is to quantitatively estimate pore aspect ratio variations in carbonate rocks by integrating the Kuster–Toksöz inclusion model, the inverse Gassmann equation, and Zimmerman pore stiffness analysis. This workflow is designed to address the limitations of previous studies that typically use fixed or assumed aspect ratio values without validating them against well-log elastic data. By applying this combined approach to a gas-bearing carbonate reservoir, the study aims to provide a more accurate representation of pore geometry and its depth-dependent evolution. The results are expected to improve the reliability of carbonate reservoir characterization and reduce uncertainties in interpreting hydrocarbon presence and rock elastic behavior.

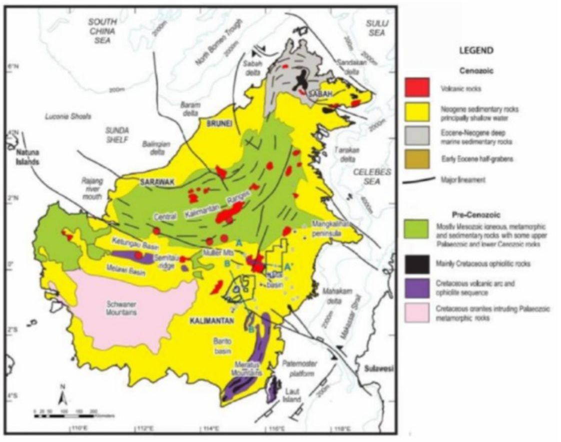

Fig 1. Regional Tectonic Map of Kalimantan

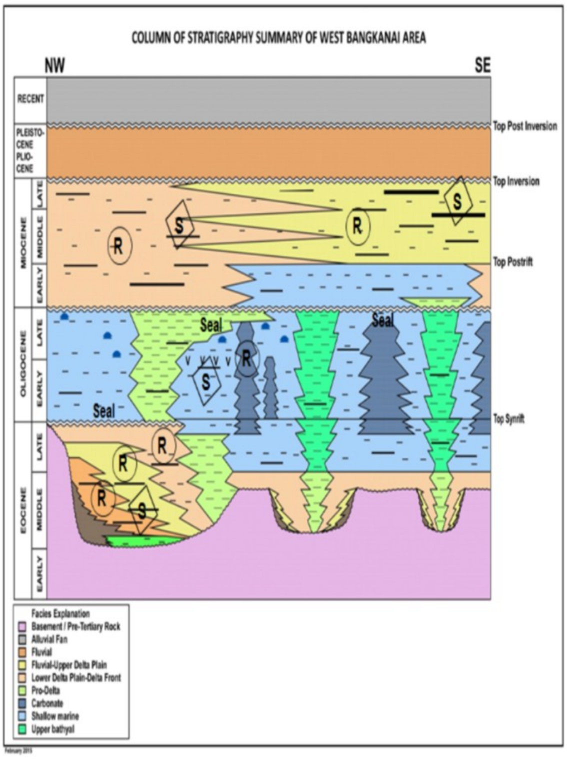

Fig 2. Stratigraphy Overview of Research Locations

2. Data

Bangkanai PSC is located within the Kutai Basin, which is bounded by the Patenoster Platform to the south, to the north by the Mangkalihat high and to the west by High Kuching (**Figure 1**).

During the Oligocene period, carbonates were developed in high basements as isolated carbonate platforms (**Figure 2**). This carbonate is the main reservoir in Bangkanai and West Bangkanai PSC and the only producer of Paleogene carbonate gas in the Kutai Basin. During the late Oligocene period extension and increase in basin margins occurred. Subsequently, uplifting of the Central Kalimantan Range gave rise to passive delta-front sedimentation that began in the Early Miocene and progressed to the east and included Kutai Carbonates.

This research was conducted on Well-1. Data available in well log data include depth log, gamma ray log, Vp log, Vs log, density log, porosity log, Vsh log, and Sw log. Rock physics modeling is done at carbonate reservoir target intervals at depths of about 2700-3000 meters from data well.

3. Methodology

Research in general is modeling rock physics based on the elastic property of rocks at the target research intervals in Well-1. The stages of research carried out in this study are as follows:

Extraction of Elastic Properties from Well Logs. P-wave velocity (Vp), S-wave velocity (Vs), and density logs were used to calculate the initial saturated bulk and shear moduli of the rock. Only intervals with water saturation (Sw = 1) were selected to avoid uncertainties related to multiphase fluids.

Estimation of Dry Frame Moduli Using the Inverse Gassmann-Equation. The Biot–Gassmann inverse formulation was applied to derive the dry bulk modulus (K_{dry}) from the saturated elastic parameters. This step provides the rock frame properties before the effect of pore fluid is introduced.

Pore Stiffness Evaluation Using the Zimmerman Model. The dry modulus values were normalized against mineral moduli (K_min) to generate pore stiffness trends following the Zimmerman equation. Three characteristic pore stiffness categories soft, reference, and stiff pores were identified as initial indicators of pore geometry.

Determination of Initial Aspect Ratio Range. Based on the Zimmerman pore stiffness results and literature constraints, an initial aspect ratio range ($\alpha = 0.01 - 0.9$) was selected to represent crack like to equant pore geometries. These values served as inputs for the inclusion modeling.

Kuster–Toksöz Inclusion Modeling (Dry Condition). The Kuster–Toksöz model was applied to compute the effective dry bulk and shear moduli for different pore aspect ratios. Inputs include mineral moduli, porosity, and the selected aspect ratio values. This enables prediction of the elastic response of the rock frame for each pore shape scenario.

Fluid Substitution Using the Biot–Gassmann Equation. The effective dry moduli from the inclusion model were then resaturated using the Biot–Gassmann equation to compute the saturated bulk modulus and corresponding Vp and Vs. A 100% water-saturated condition (Sw = 1) was applied to maintain consistency with the well log calibration interval.

Velocity Model Calibration Against Measured Logs. The modeled P-wave and S-wave velocities were compared with measured log data at each depth. The best-fitting aspect

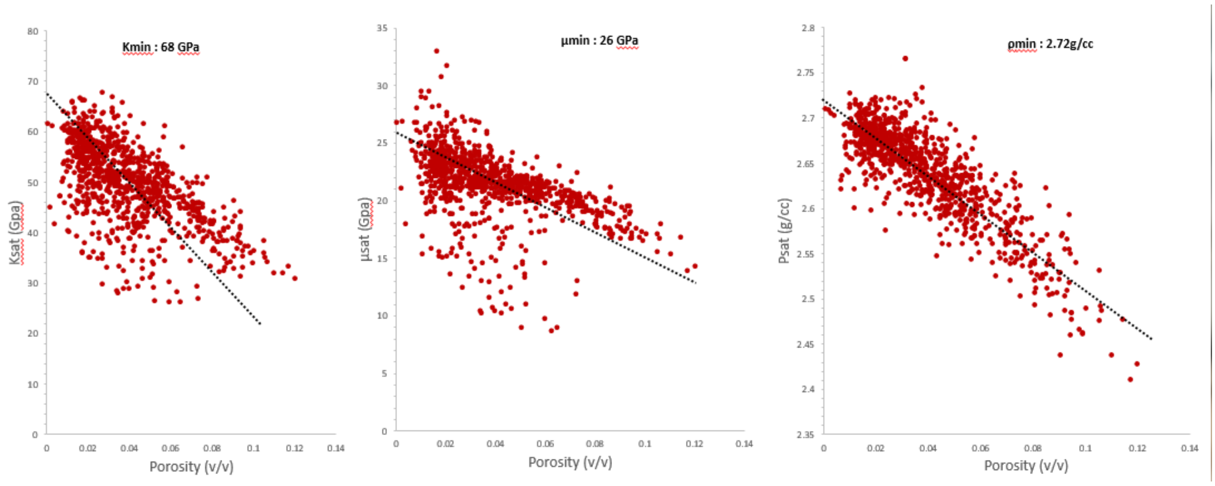

Fig 3. Cross-plots of porosity versus elastic mineral parameters used to extrapolate the values at zero porosity, representing the solid mineral moduli. From left to right, the plots correspond to bulk modulus, shear modulus, and density.

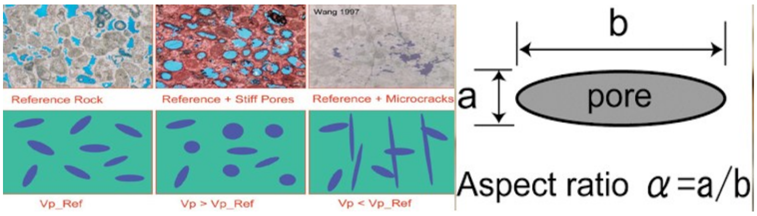

Fig 4. The aspect ratio in the spherical pore shape and illustration of different pore types in carbonate (Wang, 1997)

ratio values were identified for each interval, allowing interpretation of pore geometry variations with depth.

The plot between porosity and elastic mineral parameters (**Figure 3**) illustrates the variation of bulk modulus, shear modulus, and density as porosity changes. As expected, all elastic parameters increase with decreasing porosity. Extrapolation to zero porosity represents the properties of the solid mineral framework, providing the reference values of bulk modulus, shear modulus, and density for the rock matrix. Specifically, the bulk modulus shows the highest sensitivity to porosity reduction, followed by the shear modulus, while density exhibits a relatively smaller change. These results confirm that the mineral framework becomes stiffer and denser as the pore space diminishes, which is essential for accurately modeling the mechanical behavior of carbonate rocks in subsequent rock physics analyses.

3.1 Biot-Gassmann Equation

The dry bulk modulus obtained from the inverse Gassmann equation was then analyzed using the Zimmerman pore stiffness model to evaluate the relationship between porosity and frame stiffness. In this step, the ratio between the dry bulk modulus (K_{dry}) and the mineral bulk modulus (K_{min}) was computed and plotted against porosity to determine the stiffness trend of the pore space. The Zimmerman model provides three characteristic pore responses soft, reference, and stiff pores which represent different pore geometries and compaction states in carbonate rocks.

This analysis allows the identification of the dominant pore behavior within the target interval and serves as the initial constraint for selecting aspect ratio values to be tested in the Kuster–Toksöz inclusion model. The pore stiffness trend derived from the Zimmerman curve is used not as a final result, but as a diagnostic tool to guide the range of aspect ratios (α) applied in the subsequent modeling steps.

The Gassmann equation can be expressed as follows:

$$K_{\text{dry}} = \frac{K_{\text{sat}}\left(\frac{\phi K_{\text{min}}}{K_{\text{fluid}}} + 1 - \phi\right) - K_{\text{min}}}{\frac{\phi K_{\text{min}}}{K_{\text{fluid}}} + \frac{K_{\text{sat}}}{K_{\text{min}}} - 1 - \phi} \tag{1}$$

where K_{sat} the saturated bulk modulus of the rock, $K_{\text{dry}}$is the dry bulk modulus of the rock, $K_{\text{matrix}}$is the bulk modulus of the solid rock matrix, $K_{\text{fluid}}$is the bulk modulus of the pore fluid, $K_{\text{min}}$is the bulk modulus of the mineral, and ϕ denotes the rock porosity.

To describe the dependence of the dry-frame stiffness on porosity, the normalized dry bulk modulus is expressed as

$$\frac{K_{\text{dry}}}{K_{\text{min}}} = \frac{1}{1 + \frac{\phi}{k}} \tag{2}$$

where k is dimensionless pore-space stiffness parameter. This parameter is defined as

$$k = \frac{K_{\phi}}{K_{\text{min}}} \tag{3}$$

with K_{ϕ} denoting the pore-space bulk modulus, which characterizes the compressibility associated with the pore geometry. Thus, k describes the stiffness of the pore space relative to the stiffness of the mineral phase.

3.2 Kuster-Toksöz Model

The Kuster-Toksöz inclusion model is a rock physics model developed by Gerald Kuster and M. Nafi Toksöz in the early 1970s. This model is particularly useful in calculating the effective elastic properties of a composite material, such as rocks with inclusions (pores, cracks, or different mineral grains) of various shapes and sizes (Liu et al., 2020). In geophysics, it is widely used to simulate the effect of pore shape and aspect ratio on the bulk and shear moduli of rocks, providing insights into the mechanical behavior of heterogeneous rocks, including carbonate and sandstone formations.

As illustrated in **Figure 4**, the model defines inclusions by their shape (spherical, ellipsoidal, or crack-like) and aspect ratio, which significantly affect the rock's overall elastic properties. For example, low-aspect-ratio inclusions (cracks) reduce the rock's stiffness more than high-aspect-ratio (spherical) inclusions. The Kuster and Toksöz equations for the inclusion of penny cracks that involve aspect ratios can be expressed as follows:

$$k_{KT} = \frac{\left(k_s + \phi \frac{\frac{4}{3}\mu_s(k_{fl} - k_s)}{k_s + \frac{4}{3}\mu_s} P^{si}\right)}{\left(1 - \phi \frac{(k_{fl} - k_s)}{k_s + \frac{4}{3}\mu_s} P^{si}\right)} \quad (4)$$

$$\mu_{KT} = \mu_s \frac{\left(1 - \phi Q^{si} \frac{\zeta_s}{\zeta_s + \mu_s}\right)}{\left(1 + \phi Q^{si} \frac{\mu_s}{\mu_s + \zeta_s}\right)} \quad (5)$$

where k_{KT} and μ_{KT} are Kuster–Toksöz bulk modulus and shear modulus, respectively. Meanwhile, $k_s$and $\mu_s$denote the bulk and shear moduli of the solid matrix, whereas k_{fl}represents the bulk modulus of the inclusion phase, which may correspond to a pore fluid or a pore-filling material. The parameter ϕdenotes the volume fraction of inclusions and is equivalent to porosity when the inclusions are pores. The terms $P^{si}$and $Q^{si}$are inclusion shape factors for bulk and shear deformation, respectively, and account for the influence of inclusion geometry and aspect ratio on the composite elastic response. The parameter $\zeta_s$is an auxiliary shear-coupling term defined from the matrix elastic moduli and governs the interaction between matrix stiffness and inclusion compliance.

4. Results and Discussion

From the initial determination of the bulk and shear modulus mineral to the modeling of pore space stiffness with the Zimmermann equation is to get the initial value as an input for the Toksöz Kuster model. The inputs involved are bulk modulus and mineral shear (K_{min}, μ_{min}), porosity (ϕ), bulk fluid modulus (K_{fl}) and aspect ratio (α).

Based on the theory of pore space stiffness by Zimmermann on **Equation 2**, Well-1 data are plotted as in **Figure 5**. The image shows the K_{dry}/K_{min} values for the porosity of Well-1 at the water saturated target interval (Sw = 1), where the minimum k value is at $k = 0.01$ (soft pore), the maximum value at $k = 0.9$ (stiff pore) and the dominant value at $k = 0.15$ (reference pore). These values are the initial guidance to see the behavior of data on pore shape variations in carbonate rocks.

Kuster-Toksöz modeling gives the value of bulk modulus of rock in dry conditions without being saturated by pore fluid. The aspect ratio fitting was performed on models ranging from 0.01 to 0.9. By using a dry rock framework, the fluid substitution is performed to recalculate the saturated bulk modulus. The fluid substitution method is carried out with the Biot-Gassmann equation under 100% water saturated conditions (Sw = 1).

Figure 6 shows the results of the velocity modeling of the rock physics model used and the measurement velocity derived from log data. It can be observed that the results of the P and S velocity modeling have a good match at a depth of 2800 m - 3000 m with an aspect ratio 0.05. This shows that the depth interval is in accordance with aspect ratio used. For depth intervals above 2800 m, it is suitable with aspect ratio of 0.03. The difference in the aspect ratio of

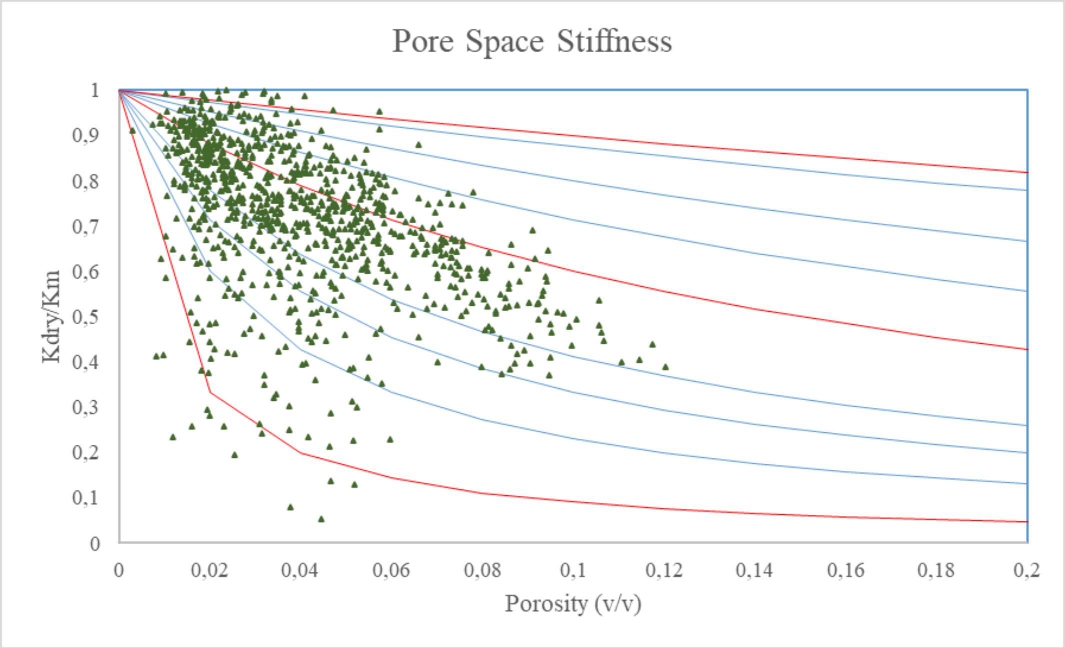

Fig 5. Results of the pore space stiffness model by Zimmermann for Well-1 data

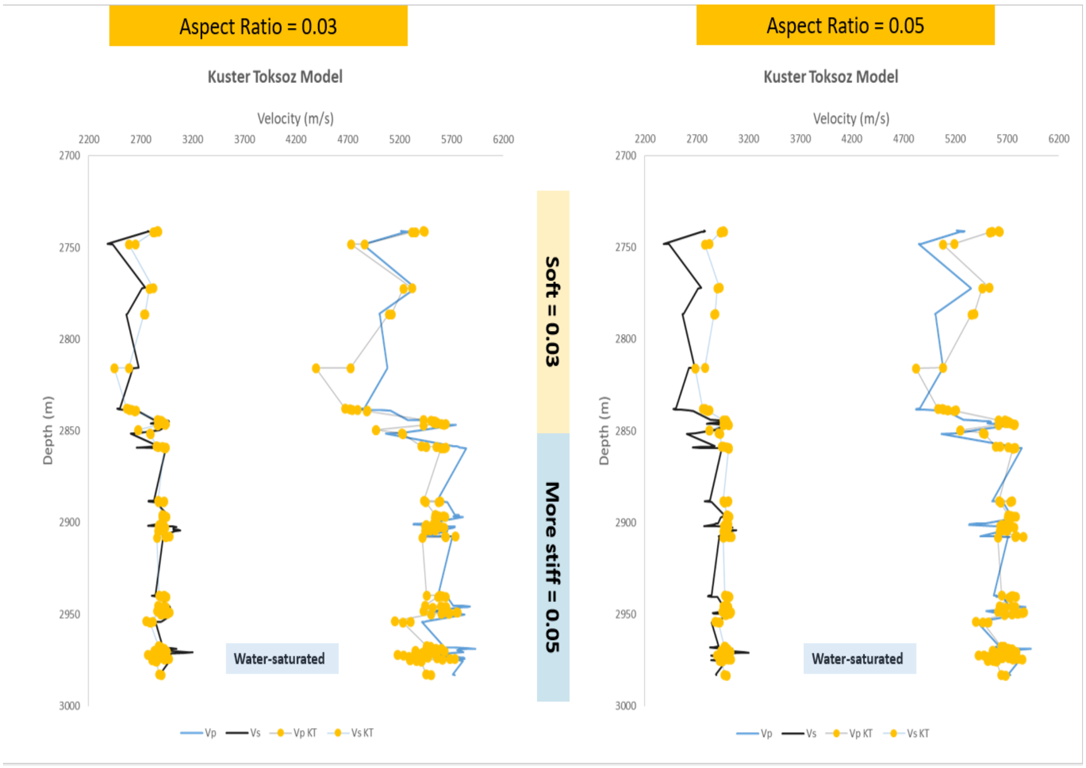

Fig 6. Fitting data between the velocity of the modeling results and the measurement velocity of the log data, P wave velocity (left) and S wave velocity (right). the model is 100% water saturated or Sw = 1

each depth is related to the compacting process of rock formation due to increased depth, which shows that the deeper the depth the rock pore will be stiffer. So, it is very important to determine the value of aspect ratio in the rock physics model under study.

5. Conclusion

Based on the results of the study, it can be concluded that:

- Carbonate rock modeling with the inclusion model especially the Kuster-Toksöz equation gives good results for the elastic property of the rock.
- Modeling using penny shape cracks represents complex pore types of carbonate rocks with certain aspect ratios. Determination of the right aspect ratio will provide optimal results for carbonate rock models.
- The combination of the Biot-Gassmann fluid substitution method makes it possible to remodel the bulk modulus of the rock in a fluid saturated state.
- The overall results of the velocity model provide information that each depth interval has a different aspect ratio value associated with the rock compacting process that changes the pore geometry.s

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