

Simpson's 3/8 Methods to Calculate Approximate Area of West Java Province

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ABSTRACT

Numerical methods play an essential role in solving complex problems that cannot be addressed analytically. This study focuses on the application of Newton–Cotes numerical integration techniques, specifically Simpson's 3/8 rule, to estimate the area of West Java Province, Indonesia. Previous studies applied the trapezoidal rule for this purpose and reported an approximation error of 3.51%. In this research, Simpson's 3/8 rule is employed due to its use of cubic polynomial interpolation, which provides higher accuracy for smooth functions. The province is divided into four quadrants, and the area is approximated using 9, 18, and 36 partitions. The results indicate that increasing the number of partitions leads to improved accuracy of the area estimation. The findings show that Simpson's 3/8 rule consistently outperforms the trapezoidal method. The smallest relative error obtained is 2.97% using 36 partitions. These results confirm the effectiveness of Simpson's 3/8 rule for approximating irregular geographic areas and demonstrate its suitability for applications in geographic analysis.

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1. INTRODUCTION

Numerical methods comprise a collection of mathematical techniques developed to address problems that are otherwise challenging or impossible to resolve analytically [1]. Their importance spans various scientific disciplines, particularly in providing practical strategies for approximating

definite integrals. Numerical integration, or quadrature, includes several foundational approaches such as the trapezoidal rule, Simpson's 1/3 rule, and Simpson's 3/8 rule, which serve as essential tools when analytical solutions are not feasible [2].

Within this domain, the Newton Cotes formulas are among the most widely adopted because they approximate integrals by fitting interpolating polynomials to subdivided segments of the integration interval [3, 4]. The Simpson family of rules differs primarily in the order of interpolation used. Simpson's 3/8 rule utilizes cubic interpolation, whereas Simpson's 1/3 rule employs quadratic interpolation [5]. These distinctions influence the accuracy and applicability of each method.

A significant advantage of Simpson-based approaches lies in their fourth-order error behavior, with the composite Simpson method yielding an error term on the order of $O(h^4)$, which generally provides greater accuracy than the trapezoidal rule [6]. This property enhances the suitability of Simpson's 3/8 rule for applications requiring precision and aligns well with recent numerical acceleration techniques that further optimize computational results [7]. Theoretical analyses confirm that this classical error bound holds for functions that are four times continuously differentiable, as the bound depends on the norm of the fourth derivative [8]. Additional studies reinforce the mathematical reliability of Simpson's 3/8 rule, often referred to as Simpson's second formula, by establishing rigorous error inequalities comparable to those associated with Simpson's 1/3 rule [9].

One well-established application of numerical integration is the approximation of irregular areas. By partitioning a region into contiguous strips, the sum of these strips' areas provides an estimate of the total region. Yafet [10] demonstrates this concept through numerical integration, although the analysis does not extend to real geographic areas. Beyond the estimation of area, numerical methods also support various applications such as solving nonlinear equations, analyzing stability in milling processes, and modeling aspects of the respiratory system [11, 12, 13]. Simpson's 1/3 rule is recognized for its high accuracy but is constrained by the requirement that the number of subintervals be even [4].

In the domain of geographic area estimation, prior work employed the trapezoidal rule with 36 partitions to approximate the area of West Java Province, resulting in a relative error of 3.51% [14]. Building upon this baseline, the present study introduces a focused comparative analysis within this specific regional context by implementing the Newton–Cotes Simpson's 3/8 rule. The distinct contribution of this research lies in benchmarking this higher-order technique against the established trapezoidal results, with particular attention to the irregular topological boundaries of West Java Province.

While Simpson's 3/8 rule imposes a stricter constraint requiring the number of subintervals to be a multiple of three, it is employed here to systematically evaluate its relative performance and convergence behavior in comparison with the trapezoidal method. Consequently, the primary objective of this study is not to determine an exact geographic area, but to quantify the reduction in approximation error and assess the extent to which Simpson's 3/8 rule improves agreement with the reference area within this regional case study.

2. RESEARCH METHOD

2.1. Determination of the area by Simpson's rule 3/8

Despite the fundamental usefulness of the Trapezoidal rule, computational studies indicate that Simpson's methods provide superior performance for complex functions, yielding results that closely match exact analytical solutions [15]. Simpson's 3/8 rule relies on cubic rather than quadratic interpolation [4]. In this method, the integration domain is partitioned into subintervals of width h ,

and within each subinterval a cubic polynomial is constructed using four uniformly spaced points. These points include the two endpoints of the subinterval and two additional points located at distances $h/3$ from each endpoint [16]. The direct and uncomplicated formulation of Simpson's 3/8 rule, in contrast to more elaborate derivative-based techniques, facilitates straightforward implementation. This simplicity enhances its suitability for direct coding and broadens its applicability in computational workflows [17].

To approximate the integral over each subinterval, the Simpson's 3/8 rule computes the area under the cubic polynomial defined by the four selected points. This is achieved by evaluating the polynomial at those points and applying the composite Simpson's 3/8 rule formula:

$$I = \int_a^b f(x) dx \approx \int_a^b f_3(x) dx \quad (1)$$

$$I \approx (b - a) \frac{f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)}{8} \quad (2)$$

Where $[a, b]$ represents the interval of integration and $f(x)$ is the integrated function. The overall approximation of the definite integral is obtained by summing the approximations of the integrals over all subintervals [18].

Simpson's 3/8 composition rule is derived by partitioning the interval $[a, b]$ into n subintervals with a uniform length of $h = (b - a)/n$. It is important to note that this rule is applicable only when the value of n is a multiple of three [19].

$$\int_a^b f(x) dx \approx \frac{3}{8}h \sum_{i=1}^{\frac{n}{3}} [f(x_{3i-3}) + 3f(x_{3i-2}) + 3f(x_{3i-1}) + f(x_{3i})] \quad (3)$$

$$\int_a^b f(x) dx = \frac{3}{8}h \left[f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + 3f(x_4) + 3f(x_5) + 2f(x_6) + \dots \right. \\ \left. + 2f(x_{n-3}) + 3f(x_{n-2}) + 3f(x_{n-1}) + f(x_n) \right] \quad (4)$$

$$\int_a^b f(x) dx = \frac{3}{8}h \left[f(x_0) + f(x_n) + 3 \sum_{\substack{i=1 \\ i \not\equiv 0 \pmod{3}}}^{n-1} f(x_i) + 2 \sum_{\substack{i=3 \\ i \equiv 0 \pmod{3}}}^{n-3} f(x_i) \right] \quad (5)$$

In this study, the area of West Java province was approximated by partitioning the region into four quadrants. For each quadrant, numerical integration was applied using the composite Simpson's 3/8 rule with varying numbers of partitions ($n = 9, 18$, and 36). The integration was performed along a predefined axis, where the function $f(x)$ represents the transverse width of the study region at a given position. The discrete $f(x)$ values used in the computations were obtained from previously published work [14] and are treated as secondary numerical data derived from digital map-based boundary measurements. All $f(x)$ values are expressed in kilometers, consistent with the original data source. The approximate areas of the individual quadrants were then aggregated to produce an overall approximation of the total area of West Java province.

2.2. Calculation of Relative Error

To assess the level of accuracy in the obtained results, it is possible to compute the relative error value using the following formula:

$$\text{Relative Error (\%)} = \left| \frac{\text{exact value} - \text{approximated value}}{\text{exact value}} \right| \times 100 \quad (6)$$

A smaller percentage of error indicates a higher level of accuracy in the analysis results.

3. RESULT AND ANALYSIS

This paper presents an innovative approach to approximating the West Java Province area using the Newton Cotes method and the Simpson 3/8 rule. West Java is a thriving province in Indonesia, covering a vast area of 35,377.75 km² [20].

To calculate the area of the West Java province, the Simpson's 3/8 Rule method can be utilized. According to the study by W. N. Fauziyah et al. [14], the West Java region is divided into four quadrants and each quadrant is further partitioned to map the irregular shape onto the axis of symmetry, enhancing the accuracy of the area calculation.

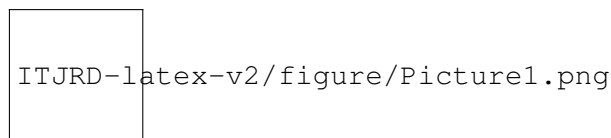


Figure 1. Division of West Java with 9 partitions for each region.

3.1. Region Calculation with 8, 18, and 36 Partitions

Figure 1 illustrates the partitioning of the West Java map into its four quadrants, with Bandung City assumed to be the central point at coordinates (0,0). For Region 1 (quadrant one), the x-axis length is 125.71 km. When partitioned into 9 segments, the partition width is calculated as 13.96 km, derived by dividing the x-axis length by the number of partitions. Dividing this region into 18 and 36 partitions results in values of 6.98 km and 3.49 km, respectively, using the same method[8].

In Region 2 (quadrant two), the x-axis length is 134.71 km. Partitioning this region into 9 segments results in a segment width of 14.96 km. For 18 and 36 partitions, the values become 7.48 km and 3.74 km, respectively[8].

For Region 3 (quadrant three), which also has an x-axis length of 134.71 km, the partition for 9 segments is again calculated as 14.96 km. When divided into 18 and 36 partitions, values are 7.48 km and 3.74 km, respectively, following the same calculation method[8]. These partitioning values are consistent with those found in region 2.

Region 4 (quadrant four) has an x-axis length of 125.71 km. When partitioned into 9 segments, the partition width is 13.96 km. For 18 and 36 partitions, the partition width is calculated to be 6.98 km and 3.49 km, respectively [8]. These values are identical to those in region 1.

3.2. Area Calculation

1. Total Area for 9 Partitions: Given $n = 9$ intervals, the Simpson's 3/8 Rule is applied as follows:

$$S_9(f) = \frac{3}{8}h \left[f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + 3f(x_4) \right. \\ \left. + 3f(x_5) + 2f(x_6) + 3f(x_7) + 3f(x_8) + f(x_9) \right]$$

(a) Area of Region 1

$$S_9(f) = \frac{3}{8}(13.96) \left[87.78 + 3(83.70) + 3(79.61) + 2(82.09) + 3(91.57) \right. \\ \left. + 3(79.15) + 2(71.77) + 3(44.16) + 3(38.54) + 48.76 \right] \\ = 8870.34105 \text{ km}^2$$

(b) Area of Region 2

$$S_9(f) = \frac{3}{8}(14.96) \left[88.08 + 3(81.41) + 3(83.49) + 2(95.11) + 3(113.50) \right. \\ \left. + 3(96.04) + 2(101.26) + 3(45.90) + 3(44.03) + 46.76 \right] \\ = 10772.2659 \text{ km}^2$$

(c) Area of Region 3

$$S_9(f) = \frac{3}{8}(14.96) \left[64.44 + 3(63.90) + 3(63.70) + 2(63.90) + 3(65.30) \right. \\ \left. + 3(68.04) + 2(70.14) + 3(68.78) + 3(24.12) + 29.47 \right] \\ = 7985.89110 \text{ km}^2$$

(d) Area of Region 4

$$S_9(f) = \frac{3}{8}(13.96) \left[64.58 + 3(66.38) + 3(74.82) + 2(79.19) + 3(77.45) \right. \\ \left. + 3(78.44) + 2(77.27) + 3(79.03) + 3(63.00) + 58.16 \right] \\ = 9177.0597 \text{ km}^2$$

(e) Total Area for 9 partitions

$$A_{\text{total}} = 8870.34105 + 10772.2659 + 7985.8911 + 9177.0597 \\ = 36805.55775 \text{ km}^2$$

2. Total Area for 18 Partitions: Given $n = 18$ intervals, the Simpson's 3/8 Rule is applied as follows:

$$S_{18}(f) = \frac{3}{8}h \left[f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + 3f(x_4) \right. \\ \left. + \cdots + 2f(x_{15}) + 3f(x_{16}) + 3f(x_{17}) + f(x_{18}) \right]$$

(a) Area of Region 1

$$\begin{aligned} S_{18}(f) &= \frac{3}{8}(6.98) \left[87.76 + 3(85.81) + 3(83.70) + 2(81.12) + 3(79.61) \right. \\ &\quad \left. + 3(78.21) + \cdots + 2(38.54) + 3(41.86) + 3(39.07) + 48.76 \right] \\ &= 8977.0827 \text{ km}^2 \end{aligned}$$

(b) Area of Region 2

$$\begin{aligned} S_{18}(f) &= \frac{3}{8}(7.48) \left[88.06 + 3(81.41) + 3(83.19) + 2(83.49) + 3(86.74) \right. \\ &\quad \left. + 3(95.11) + \cdots + 2(45.82) + 3(44.03) + 3(43.02) + 46.28 \right] \\ &= 10556.02845 \text{ km}^2 \end{aligned}$$

(c) Area of Region 3

$$\begin{aligned} S_{18}(f) &= \frac{3}{8}(7.48) \left[64.44 + 3(62.40) + 3(63.90) + 2(64.68) + 3(63.70) \right. \\ &\quad \left. + 3(64.31) + \cdots + 2(25.06) + 3(24.12) + 3(27.45) + 29.47 \right] \\ &= 7902.5826 \text{ km}^2 \end{aligned}$$

(d) Area of Region 4

$$\begin{aligned} S_{18}(f) &= \frac{3}{8}(6.98) \left[64.58 + 3(65.43) + 3(66.38) + 2(70.65) + 3(74.82) \right. \\ &\quad \left. + 3(75.75) + \cdots + 2(63) + 3(76.69) + 3(68.81) + 58.16 \right] \\ &= 9218.78265 \text{ km}^2 \end{aligned}$$

(e) Total Area for 18 partitions

$$\begin{aligned} A_{\text{total}} &= 8977.0827 + 10556.02845 + 7902.5826 + 9218.78265 \\ &= 36654.4764 \text{ km}^2 \end{aligned}$$

3. Total Area for 36 Partitions: Given $n = 36$ intervals, the Simpson's 3/8 Rule is applied as follows:

$$\begin{aligned} S_{36}(f) &= \frac{3}{8}h \left[f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + 3f(x_4) + 3f(x_5) \right. \\ &\quad \left. + \cdots + 2f(x_{33}) + 3f(x_{34}) + 3f(x_{35}) + f(x_{36}) \right] \end{aligned}$$

(a) Area of Region 1

$$\begin{aligned} S_{36}(f) &= \frac{3}{8}(3.49) \left[87.76 + 3(86.26) + 3(85.81) + 2(84.21) + 3(83.70) \right. \\ &\quad \left. + 3(81.8) + \cdots + 2(38.66) + 3(39.07) + 3(42.78) + 48.76 \right] \\ &= 8910.519674999998 \text{ km}^2 \end{aligned}$$

(b) Area of Region 2

$$\begin{aligned} S_{36}(f) &= \frac{3}{8}(3.74) \left[88.06 + 3(84.63) + 3(81.41) + 2(81.67) + 3(81.41) \right. \\ &\quad \left. + 3(83.32) + \cdots + 2(45.72) + 3(43.02) + 3(42.76) + 46.28 \right] \\ &= 10465.59525 \text{ km}^2 \end{aligned}$$

(c) Area of Region 3

$$\begin{aligned} S_{36}(f) &= \frac{3}{8}(3.74) \left[64.44 + 3(62.25) + 3(62.4) + 2(63.51) + 3(63.9) \right. \\ &\quad \left. + 3(64.56) + \cdots + 2(26.82) + 3(27.45) + 3(30.05) + 29.47 \right] \\ &= 7832.205150000002 \text{ km}^2 \end{aligned}$$

(d) Area of Region 4

$$\begin{aligned} S_{36}(f) &= \frac{3}{8}(3.49) \left[64.58 + 3(65.12) + 3(65.43) + 2(65.76) + 3(66.38) \right. \\ &\quad \left. + 3(68.77) + \cdots + 2(60.4) + 3(58.81) + 3(58.38) + 58.16 \right] \\ &= 9219.30615 \text{ km}^2 \end{aligned}$$

(e) Total Area for 36 partitions

$$\begin{aligned} A_{\text{total}} &= 8910.519675 + 10465.59525 + 7832.20515 + 9219.30615 \\ &= 36427.626225 \text{ km}^2 \end{aligned}$$

3.3. Calculation Area of Province West Java

The application of the Simpson's 3/8 Rule method with 9, 18, and 36 partitions in each quadrant to calculate the area of the province of West Java yielded the results presented in Table 1. As evidenced by Table 1, the approximate total area approaches the reference area as the number of partitions increases.

Table 1. Total Area Comparison Using Simpson's 3/8 Rule

Partitions	Region 1 (km ²)	Region 2 (km ²)	Region 3 (km ²)	Region 4 (km ²)	Total Area (km ²)	Reference Area (km ²)	Error (%)
9	8,870.34	10,772.27	7,985.89	9,177.06	36,805.56	35,377.76	4.04
18	8,977.08	10,556.03	7,902.58	9,218.78	36,654.48	35,377.76	3.61
36	8,910.52	10,465.60	7,832.21	9,219.31	36,427.63	35,377.76	2.97

The numerical comparison indicates that Simpson's 3/8 rule provides consistently lower estimation error than the trapezoidal method across all tested partition levels. With $n = 9$ subintervals, Simpson's 3/8 rule yielded an estimated area of 36,805.56 km² with an error of 4.04%, whereas the trapezoidal method produced a substantially larger estimate of 41,103.26 km² and an error of 13.92% [14]. For $n = 18$ subintervals, Simpson's 3/8 rule resulted in 36,654.48 km² with a reduced error

of 3.61%, while the trapezoidal method reported an area of 37,012.64 km² with an error of 4.62% [14]. Increasing the resolution to $n = 36$ further improved accuracy: Simpson's 3/8 rule yielded 36,427.63 km² with an error of 2.97%, compared to the trapezoidal estimate of 36,620.82 km² and an error of 3.51% [14].

According to numerical integration theory, reducing the step size h generally leads to a decrease in truncation error; however, this improvement is not linear and exhibits a law of diminishing returns [18]. This behavior is clearly reflected in the numerical results presented in Table 1 for Simpson's 3/8 rule. When the number of partitions increases from 9 to 18, the relative error decreases from 4.04% to 3.61%, indicating a substantial reduction in the dominant truncation error term due to the refinement of h . In contrast, further refinement from 18 to 36 partitions yields a less pronounced improvement, with the error decreasing from 3.61% to 2.97%, as the leading error components have already been substantially reduced and higher-order terms become dominant. Overall, increasing the partition number from 9 to 36 reduces the error by 1.07%, demonstrating the cumulative effect of step-size refinement.

A similar convergence trend is observed for the trapezoidal rule reported in W. N. Fauziyah et al. [14], where the relative error decreases markedly from 13.92% (9 partitions) to 4.62% (18 partitions) and further to 3.51% (36 partitions). Over the same refinement from 9 to 36 partitions, the trapezoidal error is reduced by 10.41 percentage points, reflecting the greater sensitivity of lower-order methods to coarse discretization. Although both methods benefit from partition refinement, Simpson's 3/8 rule consistently yields lower errors for the same partition configurations. Notably, the accuracy achieved by Simpson's 3/8 rule with only 9 partitions (4.04%) is comparable to that obtained by the trapezoidal rule with 36 partitions (3.51%), highlighting the superior efficiency of higher-order Newton–Cotes formulas.

The rate at which numerical integration errors decrease is governed by the order of the method. The trapezoidal rule is characterized by a second-order truncation error $O(h^2)$, whereas Simpson's 3/8 rule belongs to the Newton–Cotes family of higher-order formulas and exhibits a fourth-order truncation error $O(h^4)$ for sufficiently smooth integrands [18]. As a consequence, higher-order methods such as Simpson's 3/8 rule achieve faster convergence as h decreases, which accounts for the consistently smaller approximation errors observed in comparison with the trapezoidal method across all partition configurations considered in this study. Overall, these results demonstrate that Simpson's 3/8 rule converges more rapidly and provides higher accuracy in approximating the area of West Java compared to the trapezoidal method.

4. CONCLUSION

This study successfully fulfilled the objectives stated in the *Introduction* by applying Simpson's 3/8 numerical integration method to approximate the irregular geographical area of the West Java Province and validating its performance. The numerical experiments conducted with partition configurations of 9, 18, and 36 subdivisions demonstrated a clear convergence trend, in which the estimated values consistently approached the reference area as the partition density increased. The smallest relative error obtained was 2.97% using 36 partitions, which confirms the theoretical premise that higher subdivision improves the accuracy of the approximation and validates the suitability of Simpson's 3/8 rule for computations in irregular domains. These findings reinforce the reliability and applicability of the method in cases where analytical integration is impractical or impossible, such as estimating areas based on geographic boundaries. Building on the demonstrated results, future research may explore higher-order Newton–Cotes formulas, non-uniform or adaptive mesh refinement, hybrid numerical–GIS computation, or integration with machine-learning-assisted geospatial modeling

to further enhance accuracy and scalability for larger or more complex regional mapping applications.

5. DECLARATION OF AI-ASSISTED TECHNOLOGIES

During the drafting process, generative AI tools were utilized strictly for linguistic refinement, including improvements to grammar, spelling, and sentence structure. The fundamental scientific analysis, the application of numerical integration to the West Java region, and all subsequent interpretations remain the original work of the authors.

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